

Collective ferromagnetism in two-component Fermi-degenerate gas trapped in finite potential

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Spin asymmetry of the ground states is studied for the trapped spin-degenerate (two-component) gases of the fermionic atoms with the repulsive interaction between different components, and, for large particle number, the asymmetric (collective ferromagnetic) states are shown to be stable because it can be energetically favorable to increase the fermi energy of one component rather than the increase of the interaction energy between up-down components. We formulate the Thomas-Fermi equations and show the algebraic methods to solve them. From the Thomas-Fermi solutions, we find three kinds of ground states in finite system: 1) paramagnetic (spin-symmetric), 2) ferromagnetic (equilibrium) and 3) ferromagnetic (nonequilibrium) states. We show the density profiles and the critical atom numbers for these states obtained analytically, and, in ferromagnetic states, the spin-asymmetries are shown to occur in the central regions of the trapped gas, and grows up with increasing particle number. Based on the obtained results, we discuss the experimental conditions and current difficulties to realize the ferromagnetic states of the trapped atom gas, which should be overcome.

I. INTRODUCTION

Recent developments of laser trapping and cooling of atomic gases, which realized the Bose-Einstein condensates (BEC) for Alkali atoms [1], have opened up new interests for condensed atomic gases with multi-components: multi-component BEC [2,3] and fermi degenerate (FD) gas [4–6], and also bose-fermi mixed gas in trapped potentials [7–10].

One of interesting physics in such systems is the phase structures in the ground states. In general, multi-component systems show a variety of phase structures and phase transitions between them. A typical example is seen in the superfluid ^3He (a system with 2×2 components of orbital and spin angular momenta), where there exist more than two condensed phases resulted from different combinations of condensed components [11].

In the condensed system of trapped atomic gases with multi-components, several studies have been done about new phases and phase transitions: the BCS states in attractive two-component fermi gas [12,13], a transition to the superfluid states in the crossover region between BCS and BEC theories [14–16], phase separations of different components in the BEC [17–19], fermi gas [20–22] and bose-fermi mixed system [23,24].

In this paper, we discuss the trapped fermi-degenerate atomic gas with two-components: $m = \pm 1/2$ (magnetic quantum number) states of spin-1/2 atoms, or two of the hyperfine states of fermionic atoms with larger total spin ($F > 1/2$). Thus, we denote these components as spin degrees of freedom (up and down components, or up and down atoms) in this paper.

When the interaction between different spin components is weak, the stable ground state of the system has a symmetric state in spin-component distributions at $T = 0$ where equal number of up and down atoms make similar fermi-degenerate distributions so that the total spin of the whole system vanishes (paramagnetic state). However, for sufficiently strong interactions between components, an asymmetric state with unequal numbers of up/down atoms can be stable because it can be energetically favorable to increase the fermi energy of one component rather than the increase of the interaction energy between up-down components. (The interaction energy between up-up/down-down components can be neglected due to the Pauli blocking effects.) We call it “collective ferromagnetic state” of two-component fermi gas.

As the terms “paramagnetic” and “ferromagnetic” suggest, this theory is related with a mechanism of the metallic magnetism, originally proposed by Bloch and developed by Stoner et.al. [25], where fermi particles are the electrons in conductive bands. It should be noted that, in the theory of metallic magnetism, a role of E_{int} is done by the Hartree-Fock exchange energy. Studies of “collective ferromagnetic states” of atom gas are also interesting as experimental testing grounds of theoretical ideas in magnetism. Recently, such ferromagnetism aroused new interests in neutron-star physics as a mechanism of the magnetar (neutron stars with strong magnetic fields) [26].

In the uniform system, such ferromagnetic states have been discussed for the atom gas of ^6Li in relation with the stability of the BCS states [12]. The ferromagnetic states have also been discussed on the trapped BEC where the

interactions between different spin components are the origin of the asymmetry [27]. In this paper, we treat the ferromagnetic states in the trapped and finite fermion system.

In the next section of this paper, we formulate a set of equations for the two-component fermi gas at $T = 0$ and the stability condition of its solutions in the Thomas-Fermi (TF) approximations. In Section III, solutions of the TF-equations are analyzed algebraically, and a critical condition for the collective ferromagnetic states is obtained. In Section IV, we show the density distributions of the two-component fermi gas and discuss paramagnetic-ferromagnetic transitions in it. In Section V, we summarize the results and discuss the experimental conditions to obtain the ferromagnetic states.

II. THOMAS-FERMI EQUATIONS FOR TWO-COMPONENT FERMI GAS

We consider a $T = 0$ system of two-component Fermi gas trapped in an isotropic harmonic oscillator potential; densities of spin up/down components are denoted by $\rho_1(r)$ and $\rho_2(r)$. To describe the ground-state behaviors of the system, we use the Thomas-Fermi approximation [12,28], where the total energy of the system is a functional of the densities:

$$E = \int d^3r \left[\sum_{\sigma=1,2} \left\{ \frac{\hbar^2}{2m} \frac{3}{5} (6\pi^2)^{\frac{2}{3}} \rho_{\sigma}^{\frac{5}{3}} + \frac{1}{2} m \omega^2 r^2 \rho_{\sigma} \right\} + g \rho_1 \rho_2 \right]. \quad (1)$$

The m and ω in (1) are the fermion mass and the oscillator frequency of the trapping potential. The last term in Eq. (1) corresponds to the interaction energy between different components of fermion, and the strength of the coupling constant g is given by $g = 4\pi\hbar^2 a/m$ where a is the s -wave scattering length. The interactions between the same components are neglected; the elastic s -wave scattering is absent because of the Pauli blocking effects and the p -wave scattering is suppressed below $\sim 100 \mu\text{K}$. In the present paper, we discuss a system of repulsive interaction, so that the parameter g should be positive ($g > 0$).

As we show later, the collective ferromagnetic states of two-component fermi gas show up in the case of large particle number ($10^6 \sim 10^{13}$), which validates the use of the Thomas-Fermi approximation. Also the validity of the approximation can be estimated from the smoothness of the mean-field potential: $V_{\text{eff}} = \frac{1}{2} m \omega^2 r^2 + g \rho_{1,2}$. As a parameter that represent its smoothness, we can take a local de Broglie wave length $\lambda(r) = \hbar/p(r)$, where $p(r)$ is a TF local momentum defined by $p(r) = \sqrt{2m(\epsilon_F - V_{\text{eff}})}$ (ϵ_F : Fermi energy). As presented in [29], a validity condition with $\lambda(r)$ is given by $f(r) \equiv |\frac{d\lambda}{dr}| \ll 1$. Evaluating $f(r)$ with the parameters given in the last section (for ^{40}K), we obtain $f(r) \lesssim 10^{-3}$ except the classical turning points. It also supports the validity of the TF approximations in the present case.

To simplify Eq. (1), we introduce the scaled dimensionless variables:

$$n_{\sigma} = \frac{128}{9\pi} \rho_{\sigma} a^3, \quad x = \frac{4}{3\pi} \frac{ar}{\xi^2}, \quad \tilde{E} = \frac{2^{18}}{3^7 \pi^6} \left(\frac{a}{\xi} \right)^8 \frac{E}{\hbar\omega}, \quad (2)$$

where $\xi = \sqrt{\hbar/m\omega}$ is the oscillator length. Using these variables, Eq. (1) becomes

$$\tilde{E} = \int d^3x \left[\sum_{\sigma=1,2} \left(\frac{3}{5} n_{\sigma}^{\frac{5}{3}} + x^2 n_{\sigma} \right) + n_1 n_2 \right]. \quad (3)$$

The Thomas-Fermi equations for the densities $n_{1,2}$ are derived from the variations of the total Energy \tilde{E} on $n_{1,2}$ with a constraint on the total particle number \tilde{N} : $\frac{\delta}{\delta n_{\sigma}}(\tilde{E} - \lambda \tilde{N}) = 0$, where \tilde{N} is the scaled total particle number defined by

$$\tilde{N} = \tilde{N}_1 + \tilde{N}_2 = \sum_{\sigma} \int d^3x n_{\sigma} = \frac{2^{13}}{3^5 \pi^4} \left(\frac{a}{\xi} \right)^6 N \sim 0.346 \left(\frac{a}{\xi} \right)^6 N. \quad (4)$$

The Lagrange multiplier λ in the variational equation is for the fermion-number constraint, and related with the scaled chemical potential $\tilde{\mu}$ through the relation $\tilde{\mu}N = \lambda\tilde{N}$; using Eq. (4), we obtain

$$\tilde{\mu} \sim 0.346 \left(\frac{a}{\xi} \right)^6 \lambda. \quad (5)$$

It should be noted that, in Eq. (3), parameters (m, ω, g) has been scaled out and no parameters are included except λ . The Lagrange multiplier λ is determined by the total fermion number \tilde{N} , so that \tilde{N} is the only parameter that determine the ground-state properties of the system.

Using Eqs. (3) and (4) for the variational equation, we obtain the TF equations:

$$n_1^{\frac{2}{3}} + n_2 = \lambda - x^2 \equiv M(x), \quad n_2^{\frac{2}{3}} + n_1 = M(x). \quad (6)$$

The stability condition for solutions of Eq. (6) can be derived from the second-order variations of the energy functional:

$$\left| \begin{array}{cc} \frac{\delta^2 \tilde{E}}{\delta n_1^2} & \frac{\delta^2 \tilde{E}}{\delta n_1 \delta n_2} \\ \frac{\delta^2 \tilde{E}}{\delta n_2 \delta n_1} & \frac{\delta^2 \tilde{E}}{\delta n_2^2} \end{array} \right| \geq 0. \quad (7)$$

Using Eq. (3), we obtain the stability condition for the present case:

$$n_1 n_2 \leq \left(\frac{2}{3} \right)^6. \quad (8)$$

III. SOLUTIONS OF THOMAS-FERMI EQUATIONS

In this section, we show solutions of coupled TF equations (6) in algebraic form and discuss their stability based on the stability condition (8).

Let's introduce variables s and t as $n_1 = s^3$ and $n_2 = t^3$, then Eq. (6) becomes

$$s^3 + t^2 = M, \quad t^3 + s^2 = M. \quad (9)$$

Making sum and difference of them, we obtain two equations equivalent with Eq. (9):

$$s^3 + t^3 + s^2 + t^2 = 2M, \quad s^3 - t^3 - (s^2 - t^2) = (s - t)(s^2 + st + t^2 - s - t) = 0. \quad (10)$$

The factorized form of the second equation gives two alternatives: 1) $s = t$ or 2) $s^2 + st + t^2 - s - t = 0$.

In the case 1), $s = t$, the first equation in (10) becomes $s^3 + s^2 = M$, which can be solved algebraically with the Caldano's formula. It includes only one positive root when $M \geq 0$:

$$s = \frac{f(M)}{6} + \frac{2}{3f(M)} - \frac{1}{3}, \quad f(M) = \left[-8 + 108M + 12\sqrt{81M^2 - 12M} \right]^{1/3}. \quad (11)$$

For this solution, the stability condition (8) gives $s \leq 2/3$, which leads to the constraint for M :

$$M = s^3 + s^2 \leq \left(\frac{2}{3} \right)^2 + \left(\frac{2}{3} \right)^3 = \frac{20}{27}. \quad (12)$$

Thus, the stable symmetric solutions exist when $0 \leq M \leq 20/27$.

Next, we take the case 2) : $s^2 + st + t^2 - s - t = 0$. Now, this and the another equation in (10) are symmetric under the exchange between s and t , so that they can be represented by elementary symmetric polynomials, $A \equiv s + t$ and $B \equiv st$:

$$A^3 - A^2 - A + M = 0, \quad A^2 - A - B = 0. \quad (13)$$

The first equation has a real positive solution:

$$A = \frac{g(M)}{6} + \frac{8}{3g(M)} + \frac{1}{3}, \quad g(M) = \left[-108M + 44 + 12\sqrt{81M^2 - 66M - 15} \right]^{1/3}. \quad (14)$$

Substituting it in (13), we obtain the solution for B :

$$B = \frac{[g(M)]^2}{36} - \frac{g(M)}{18} + \frac{4}{9} - \frac{8}{9g(M)} + \frac{64}{9[g(M)]^2}. \quad (15)$$

The s and t can be recovered as two solutions of the equation: $x^2 - Ax + B = 0$. These solutions are generally asymmetric ($s \neq t$).

We can check the stability of the asymmetric solutions with Eq. (8), which gives a constraint for B : $B = st = (n_1 n_2)^{1/3} \leq 4/9$. Then, from the second equation in (13), we obtain that for A : $1 \leq A \leq 4/3$. Differentiating the first equation in (13) by A , we find that M is a monotonically decreasing function of A within the interval $1 \leq A \leq 4/3$. Combining these results, we obtain the stability range of M for the asymmetric solutions: $20/27 < M \leq 1$.

In summary, the stable solutions of Eq. (10) are

1. $0 \leq M \leq \frac{20}{27}$ symmetric,
 2. $\frac{20}{27} < M \leq 1$ asymmetric,
 3. $1 < M$ (no stable solutions).
- (16)

In case that $1 < M$, Eq. (10) have no stable solutions. In the TF variational equation $\frac{\delta}{\delta n_\sigma}(\tilde{E} - \lambda \tilde{N}) = 0$, we assumed the chemical equilibrium between fermion 1 and 2, and put their chemical potentials (Lagrange multipliers) equal: $\lambda = \lambda_1 = \lambda_2$. Accordingly, in $1 < M$, we should say that the system has no stable solutions in “equilibrium”. Thus, for $1 < M$, we should take the nonequilibrium states where all fermions occupy one component (complete asymmetry). If we assume $s = 0$, t is obtained by solving the equation:

$$t^2 = M, \quad s = 0. \quad (17)$$

IV. DENSITY PROFILES OF TWO-COMPONENT FERMI GAS

Using the results in the previous section, we can calculate the density profiles of the two-component fermi gas for any values of λ . Corresponding to the classification by $M = \lambda - x^2$ in (16), we should divide λ into three regions:

$$(i) \ 0 \leq \lambda \leq \frac{20}{27}, \quad (ii) \ \frac{20}{27} < \lambda \leq 1, \quad (iii) \ 1 < \lambda, \quad (18)$$

and discuss qualitative profiles of the density distributions $n_{1,2}$.

- (i) *Paramagnetic ground states.* In these cases, $M(x)$ satisfies $M(x) = \lambda - x^2 \leq 20/27$ for any value of x . Thus, from (16), the density distributions are composed of the symmetric solution (11) in all spatial region. Consequently, the ground states are paramagnetic in the region (i): $n_1(x) = n_2(x)$. In Fig. 1(a), the density profiles are shown when $\lambda = 1/2$. They monotonically decreases with the scaled radial distance x , and vanishes at the TF cut-off: $x_{TF} \equiv \sqrt{\lambda} = 1/\sqrt{2}$ in this case. These states occurs in the case of small fermion number.
- (ii) *Ferromagnetic ground states in equilibrium.* In these cases, we obtain $M(x) \leq 20/27$ in the outside region ($x \geq \sqrt{\lambda - 20/27}$), but $20/27 < M(x) \leq 1$ in the inside region ($x < \sqrt{\lambda - 20/27}$). Correspondingly, the density distributions become symmetric in the outside region and asymmetric in the inside region, so that the ground states become ferromagnetic in the region (ii). Because the condition $M(x) \leq 1$ is satisfied, solutions are in equilibrium in all spatial regions, and densities are partially asymmetric in the inside region. In Fig. 1(b), the density profiles are shown when $\lambda = 4/5$ for n_1 (solid line) and n_2 (dotted line). The border between symmetric and asymmetric regions is given by $x_{AS} \equiv \sqrt{\lambda - 20/27} = \sqrt{8/135}$ in this case. In the outside (symmetric) region, the both lines overlap and vanishes at $x_{TF} = 2/\sqrt{5}$.
- (iii) *Ferromagnetic ground states in nonequilibrium.* In these cases, density profiles are also asymmetric in the inside region ($x < x_{AS}$) as in the above region. However, in the most inside region ($x < \sqrt{\lambda - 1}$), $M(x) > 1$ is satisfied and, according to the condition 3 in (16), the density profiles become nonequilibrium and are given by complete asymmetric solutions in (17). In Fig. 1(c), we show the density profiles when $\lambda = 3/2$ for n_1 (solid line) and n_2 (dotted line). In the most inside region ($x < x_{EQ} = \sqrt{\lambda - 1} = 1/\sqrt{2}$), the density profiles are completely asymmetric ($n_2 = 0$), and, in $x_{EQ} \leq x < x_{AS} = \sqrt{41/54}$, they are partially asymmetric. In the outside (symmetric) region ($x > x_{AS}$), two lines overlap and vanishes at $x = x_{TF} = \sqrt{3/2}$. Thus, the ground states are ferromagnetic and in nonequilibrium in the region (iii) which corresponds to the cases of large fermion number.

In Fig. 2, we show the dependence of λ on the fermion number \tilde{N} , which is calculated by Eq. (4) using the fermion densities $n_{1,2}(x)$ obtained above; for the ground states (solid line) and the paramagnetic states (dotted line). From the above discussions, for $\lambda \leq 20/27$, the ground states are paramagnetic, so that the both lines overlap. As can be read off in this figure, the critical value $\lambda = 20/27$ corresponds to $\tilde{N}_C = 0.53$, which is the critical fermion number for the transition between paramagnetic and ferromagnetic states.

In Fig. 3(a), the variations of the scaled total energies \tilde{E} are shown against \tilde{N} , for the ground (solid line) and paramagnetic (dotted) states. The \tilde{E} can be obtained by Eq. (3) as a function of λ using the fermion densities $n_{1,2}(x)$; combined with the \tilde{N} -dependence of λ (shown in Fig. 2), we can obtain its \tilde{N} -dependence. In $\tilde{N} \leq \tilde{N}_C = 0.53$, both lines overlap completely because the ground states become paramagnetic. In Fig. 3(b), we plot the energy difference between the ground and paramagnetic states. In this figure, we can find that, in $\tilde{N} > \tilde{N}_C$, the ground-state energy (solid line) shifts lower than that of the paramagnetic states (dotted line); it shows that the ferromagnetic ground states become more stable in this region.

As can be seen in Fig. 3(a) when $\tilde{N} > 0.53$, the energy difference between ferromagnetic and paramagnetic states $|\tilde{E} - \tilde{E}_{\text{para}}|$ is very small in comparison with the total energy \tilde{E} : e.g. $|\tilde{E} - \tilde{E}_{\text{para}}|/\tilde{E} \sim 0.01$ at $\tilde{N} = 2.0$. A large part of \tilde{E} consist of the stacked kinetic energy due to the fermi degeneracy of fermions. Roughly speaking, its size can be estimated from the total energy of the noninteracting fermi gas ($g = 0$) trapped in the same harmonic oscillator potential; $\tilde{E}_{\text{non}} = \frac{3}{16}\pi^2(\frac{4}{\pi^2}\tilde{N})^{4/3}$, which increases rapidly with increasing \tilde{N} . The \tilde{E}_{non} is plotted also in Fig. 3(a) (dot-dashed line), the amounts of which are almost $\sim 75\%$ of \tilde{E} .

However, to evaluate the scale of the energy difference, we should compare it with an one-particle excitation energy at the Fermi surface, which can be estimated from the scaled chemical potential $\tilde{\mu}$ defined in (5). In case of ^{40}K with the harmonic oscillator frequency $\omega = 1000\text{Hz}$ (for other parameters, see the next section), Eq. (5) becomes $\tilde{\mu} \sim 10^{-13}\lambda$. When $\tilde{N} \sim 2$ ($\lambda \sim 1$ from Fig. 2), the ratio $\Delta\tilde{E}/\tilde{\mu}$ becomes 10^{13} . Thus, we should say that the energy difference between ferromagnetic and paramagnetic states are fairly large.

In Fig. 4, we show the fermion number asymmetry $(\tilde{N}_1 - \tilde{N}_2)/\tilde{N} = (N_1 - N_2)/(N_1 + N_2)$ against the total fermion number \tilde{N} .

V. SUMMARY AND DISCUSSIONS

We discussed the possibility of transition to ferromagnetic states in the two-component fermi gas using the Thomas-Fermi approximation from theoretical points of view. Based on the results that we obtained, let's discuss experimental conditions and also difficulties to observe ferromagnetic ground states in the trapped atomic gas. We hope that these difficulties are overcome in future developments in experimental technics.

As shown in the previous section, the ferromagnetic ground states become stable when $\tilde{N} \gtrsim 0.53$; using Eq. (4), the unscaled critical number N_C becomes

$$N_C \sim \frac{0.53}{0.346} \left(\frac{\xi}{a}\right)^6 = 1.5 \left(\frac{\xi}{a}\right)^6. \quad (19)$$

As an example, we take the ^{40}K atoms (mass $m = 0.649 \times 10^{-25}\text{kg}$) trapped in the harmonic oscillator potential with $\omega = 1000\text{Hz}$. For the scattering length, we take the value $a = 169a_B$ (a_B : Bohr radius) given in [30]. Using these parameters, we obtain $N_C \sim 10^{13}$. In recent experiments, the trapped Fermi-degenerate gas has been performed up to $\sim 10^6$ atoms, so that the larger trapping potential are necessary for realization of the ferromagnetic ground state than currently used one. In addition, because of the high central density ($\sim 10^{17}\text{cm}^{-3}$) for $\sim 10^{13}$ atoms, the inelastic/multi-body scattering processes in them becomes important, which might destroy the trapped atoms before they reach the required density.

There can be several possibilities for the reduction of N_C . For example, if the scattering length can be increased by the Feshbach resonance for ^{40}K , which has been observed experimentally [6], the value of N_C decreases and the ferromagnetic ground states can be obtained in small fermion number: simple estimation gives, $a = 820a_B$ for $N_C \sim 10^9$ and $a = 2600a_B$ for $N_C \sim 10^6$. However, in current experiments, the tuning into Feshbach resonances is done by applying the magnetic field, which should make the energy difference between the up and down states and makes the ferromagnetic transition into a crossover in the spin asymmetry. The ways to tune Feshbach resonances by non-magnetic external forces (e.g. electric fields) are preferred for the present purpose.

The use of heavier elements, e.g. Sr or Yb, is also effective for the ferromagnetic states. We hope that the combination of these methods may lead to the experimental achievements.

We also comment on the process to perform the ferromagnetic states. There exist two possibilities:

- a) The experiment starts with the magnetic field in some direction which makes the spin asymmetry (e.g. $N_1 > N_2$). Then, the magnetic field is switched off adiabatically in the cooling process. When the number of the remaining atoms is enough large, it shows the ferromagnetic states.
- b) The experiment starts with the symmetric trap ($N_1 = N_2$), and, after some relaxation time elapses, the atoms release their spin angular momenta and become the ferromagnetic state.

The case a) is similar with a standard process for observing the phase transition in the ferromagnetic materials. In case b), the spin relaxation time is considered to be the same order with that of clusterization, so that it might be difficult to observe the ferromagnetic transition within the lifetime of the atomic gas.

Finally, we comment on the spatially phase-separated states in the trapped fermionic gas in the case of the large particle number and interaction strength [20–22]. They correspond to the phase separation in the uniform system (two-phase coexistent region), discussed in [12]. The naive TF calculations give smaller energy for the ferromagnetic states, but the energy difference is very small. The competition/coexistence of these states should be an interesting problem in future.

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FIG. 1. Density profiles of fermions, n_1 (solid line) and n_2 (dotted line) for (a) $\lambda = 0.5$, (b) 0.8 and (c) 1.5. The scaled densities $n_{1,2}$ and the scaled radial distance x are dimensionless and defined by Eq. (2). The λ is a (dimensionless) Lagrange multiplier. The (a)-(c) correspond to a paramagnetic state (i), a ferromagnetic state in equilibrium (ii), a ferromagnetic state in nonequilibrium (iii) each other. The scaled fermion number $\tilde{N}_{1,2}$ are $\tilde{N}_1 = \tilde{N}_2 = 0.089$ for (a), $(\tilde{N}_1, \tilde{N}_2) = (0.33, 0.31)$ for (b), and $(3.50, 0.68)$ for (c).

FIG. 2. Lagrange multiplier λ against the scaled fermion number \tilde{N} . The solid and dotted lines are for the ground and paramagnetic states. The λ is introduced as a Lagrange multiplier for fermion-number constraint, and the \tilde{N} is defined by Eq. (4). Both quantities are dimensionless.

FIG. 3. (a) The scaled total energy \tilde{E} against the scaled total fermion number $\tilde{N} = \tilde{N}_1 + \tilde{N}_2$: the solid line is for the ground states and the dotted one for the paramagnetic states. The dash-dotted line is for the total energy of the noninteracting fermionic system, $\tilde{E}_{\text{non}} = \frac{3}{16}\pi^2(\frac{4}{\pi^2}\tilde{N})^{4/3}$. (b) The energy differences from that of the paramagnetic states, $\tilde{E} - \tilde{E}_{\text{para}}$. The solid and dotted lines are for the ground and paramagnetic states.

FIG. 4. Variation of the fermion number asymmetry $(\tilde{N}_1 - \tilde{N}_2)/\tilde{N}$ against the scaled total fermion number \tilde{N} . The solid and dotted lines are for the ground and paramagnetic states each other.